Clustering a medieval social network by SOM using a kernel based distance measure

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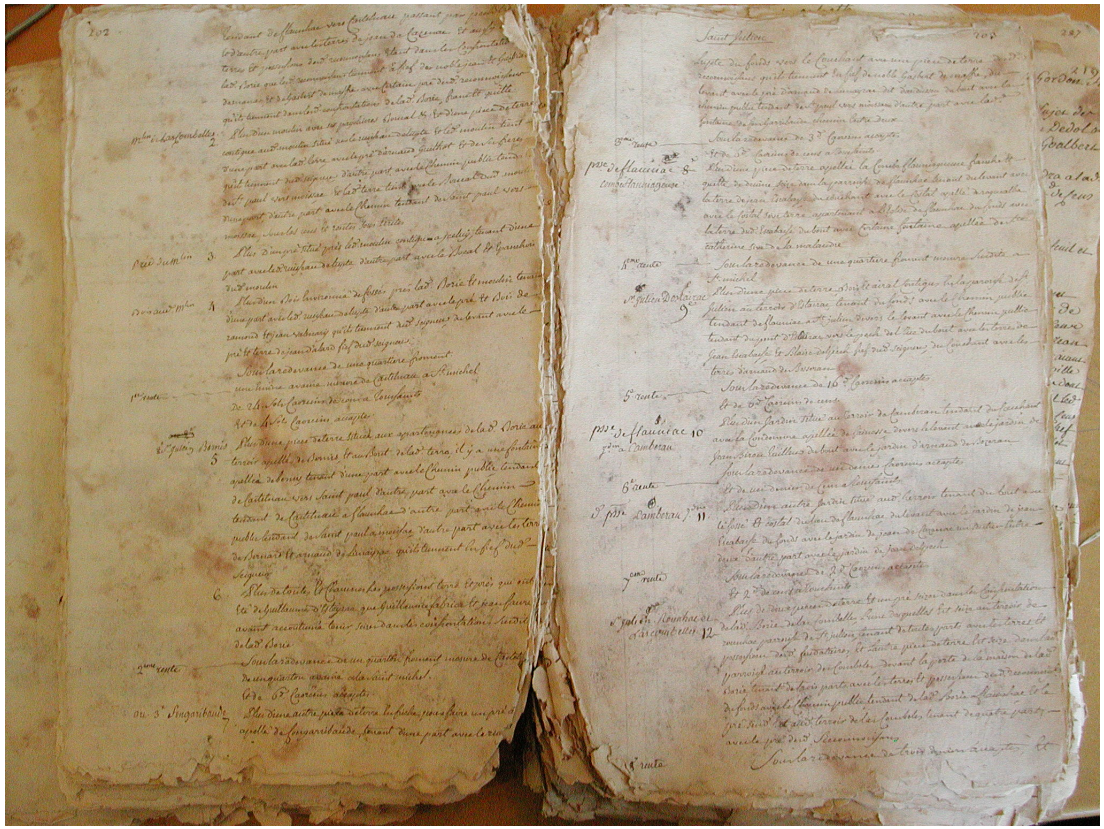
Exploring a big historic database

Data

10 000 agrarian contracts from four seignories of South West of France established between 1240 and 1520.



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- central people having a main social role ?
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First step

Restriction to a part of this database: the Castelnau-Montratier Seignory before the Hundred Years' War.

A graph clustering problem

From the database, building a **weighted graph**:

- with 226 **vertices** $x_1, \ldots, x_n$:= peasants found in the contracts;

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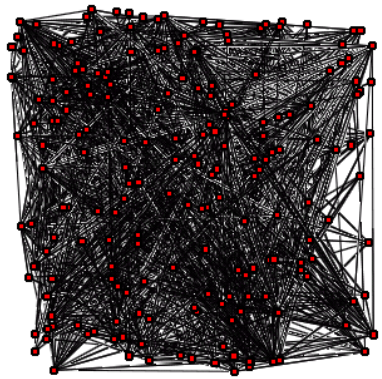

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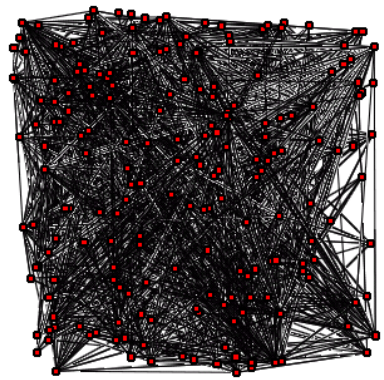

Clustering the vertices to understand the social links between peasants.

Previous work

In **[Boulet & Jouve, 2007]**, algebraic study of the non-weighted graph:

- **small world** graph $\Rightarrow$ **small number of edges between two vertices**: diameter = 5 and 90% of the couples of vertices have less than 3 edges between them;

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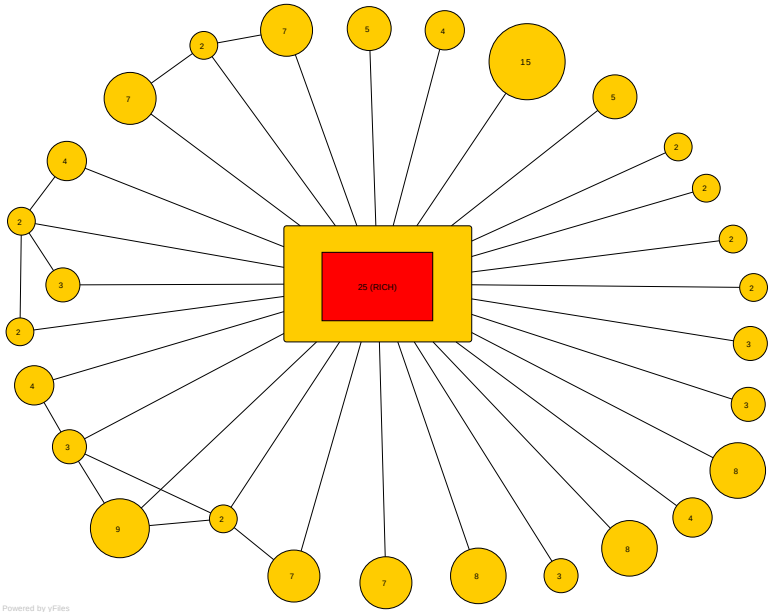
- **small world** graph $\Rightarrow$ **small number of edges between two vertices**: diameter = 5 and 90% of the couples of vertices have less than 3 edges between them;
- classification into **communities** (cliques having the same neighbors) and **rich club** (set of vertices having high degree and a high density):



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Dissimilarity SOM

[Kohohen & Somervuo, 1998], **[El Golli *et al.*, 2006]**: batch SOM for data $x_1, \ldots, x_n$ known through a dissimilarity measure δ:

1. **Initialization**: for all neurons of the grid, $j = 1, \ldots, M$, select a prototype (randomly), m_j in the dataset;
2. **Assignment**: for all data $i = 1, \ldots, n$, x_i is assigned to the neuron $f(x_i)$ such that
$$f(x_i) = \arg \min_{j=1,\ldots,M} \delta(x_i, m_j)$$
3. **Representation**: for all neuron $j = 1, \ldots, M$, m_j is re-computed:
$$m_j = \arg \min_{x \in (x_k)_{k=1,\ldots,n}} \sum_{i=1}^{n} h(f(x_i), j)\delta(x_i, x)$$
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$\Rightarrow$ Choosing an appropriate δ.

Laplacian and diffusion kernel

Definitions

For a graph with vertices $V = \{x_1, \ldots, x_n\}$ having positive weights $(w_{i,j})_{i,j=1,\ldots,n}$ such that, for all $i, j = 1, \ldots, n$, $w_{i,j} = w_{j,i}$ and $d_i = \sum_{j=1}^{n} w_{i,j}$,

- **Laplacian**: $L = (L_{i,j})_{i,j=1,\ldots,n}$ where

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$\Rightarrow$

$$\begin{array}{rccc} k^\beta : & V \times V & \to & \mathbb{R} \\ & (x_i, x_j) & \to & K^\beta_{i,j} \end{array}$$

is the **diffusion kernel** (or heat kernel).

Properties

1. **Diffusion on the graph**: $k^{\beta}(x_i, x_j) \simeq$ quantity of energy accumulated in x_j after a given time if energy is injected in x_i at time 0 and if diffusion is done along the edges.
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$\Rightarrow \delta(x_i, x_j) = \sqrt{k^\beta(x_i, x_i) + k^\beta(x_j, x_j) - 2k^\beta(x_i, x_j)}$ is a distance on $\mathcal{H} \rightsquigarrow$ dissimilarity between vertices of the graph.

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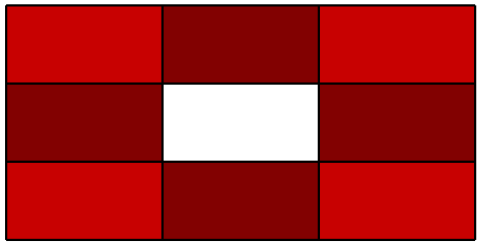
Methodology

1. **SOM**: 3×3 rectangular grid;



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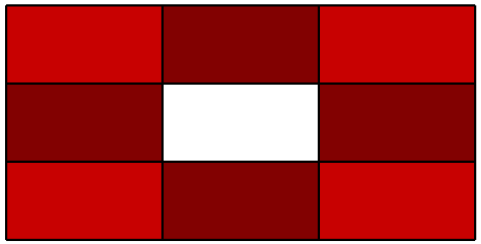

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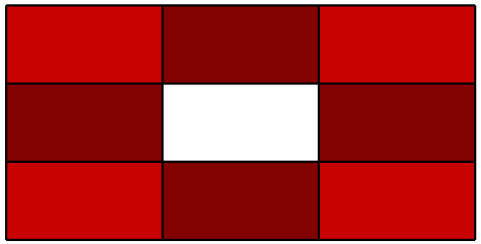

2. **Dissimilarity measure**: $\beta = 0.05, 0.1, 0.2$;
3. **Initialization**: for each β,
   - 50 random initialization;
   - algorithm ran until stabilization;
   - best classification minimizes

$$\mathcal{E} = \sum_{j=1}^{M} \sum_{i=1}^{n} h(f(x_i), j)\delta(x_i, m_j).$$

Results

31 35 69 70 79 118 155 156 159 161 175 176 198 220 246 249 272 278 281 284 285 291 316 352 *354* 384 389 396 405 432 433 434 435 442 447 449	66 94 95 96 124 125 126 132 138 142 149 240 241 245 375 *398* 418 419	25 61 64 74 *144* 201 236 247 255 412
8 10 78 119 *127* 128 131 134 135 153 181 183 184 193 200 222 237 251 258 264 276 377 386 390 394 416 420	133 *139* 150 227 262 415	9 67 129 130 *136* 137 140 141 146 148 151 152 217 260 263 413
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Synthesis map

Comparison with previous work

Class 6		Class 7
Class 4	Class 3	Class 2
	Class 5	Class 1

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Class 6		Class 7
Class 4	Class 3	Class 2
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Similarities with historian's prior knowledge

- Class 1 & Class 2 : homogeneous geographical settings (Castrum de Flaugnac & Castrum de La Graulière);
- But "Combelcau" family is not in Class 1 although part of them live in Castrum de Flaugnac;
- "Combelcau" family constitutes a large part of Class 2.

$\Rightarrow$ Family links are more important than geographical ones.

Comparison with previous work

Class 6		Class 7
Class 4	Class 3	Class 2
	Class 5	Class 1

Similarities with algebraic work on the non-weighted graph

- Each class corresponds to one or several connected communities;
- Class 3 is a part of the “rich club” and “Combelcau” family plays a great role both in Class 3 and in the rich club;
- Star-shaped structure around class 3 is similar to star-shaped structure around the rich club.

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Conclusion

Expected developments:

- Application to the whole database;
- Find a criterium to select an optimal β;
- Find an automatic procedure to combine/compare several maps obtained for different β.

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