Data analysis for labeled graphs

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Joint work with **Thibault Laurent** (Toulouse School of Economics)

Collaboration

Network analysis
(social, biological...)

Spatial statistics
(R package “GeoXp”)

Plan

Notations and examples

Data: A weighted undirected **network** modeled by a graph $\mathcal{G}$ with n nodes $x_1, \ldots, x_n$ with **weight matrix** W: $W_{ij} = W_{ji}$ and $W_{ii} = 0$.

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where c_j is either a numerical information or a factor information.

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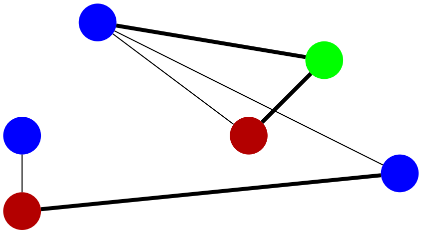

Examples: Gender in a social network, Functional group of a gene in a gene interaction network...

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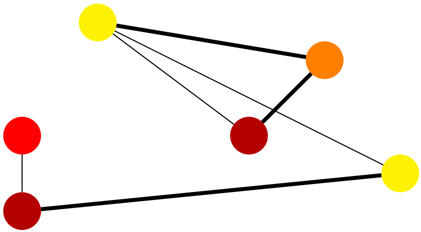
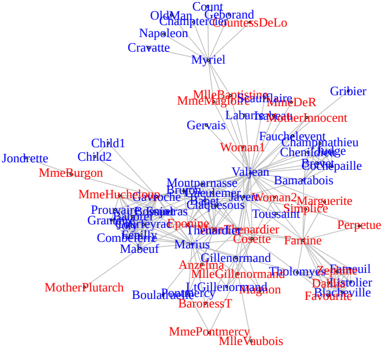
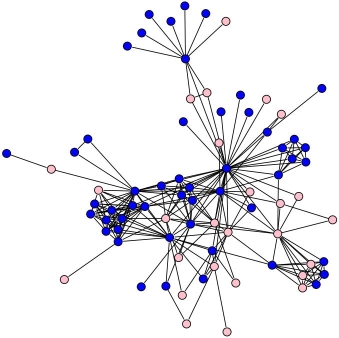


Examples: Weight of people in a social network, Number of visits of a web page in WWW...

"Real world" examples

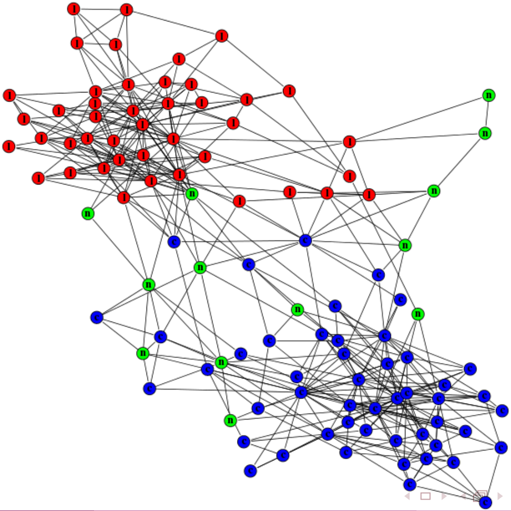
Example 1: Co-appearance network of the novel "Les Misérables" (Victor Hugo) where the nodes are labeled with gender (F/M).





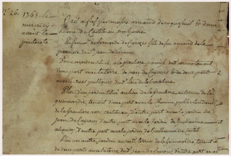
"Real world" examples

Example 2: Co-purchase network: nodes are books sold by "Amazon" and are labeled according to the political orientation of the book



"Real world" examples

Modeling a large corpus of medieval documents



Notarial acts (mostly **baux à fief**, more precisely, land charters) established in a **seigneurie** named "Castelnau Montratier", written between 1250 and 1500, involving tenants and lords.[a]

[a] http://graphcomp.univ-tlse2.fr

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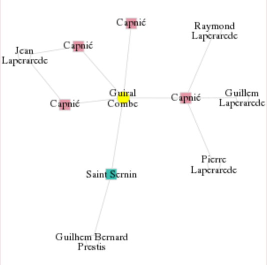

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- edges: an individual is directly involved in a transaction (6 455 edges)
- labels (transactions only): location (parish)

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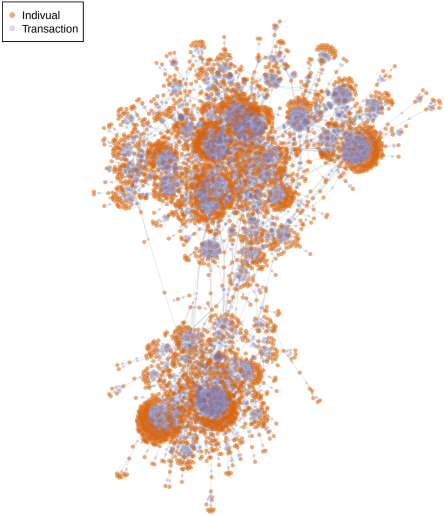

Questions?

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Are the nodes labeled with a given value **more connected to nodes with the same value** than expected? less connected?
where *"expected"* means: in comparison to a random distribution over the network.

First approach: Use of "spatial" indexes

[Laurent and Villa-Vialaneix, 2011], by identifying

- the spatial matrix (in spatial data)
- the adjacency matrix (in network)

calculate

$$JC = \frac{1}{2} \sum_{i \neq j} W_{ij} \xi_i \xi_j$$

and a MC permutation test helps measuring the strength of the link between the labels and the network structure.

A toy example: “Les Misérables”

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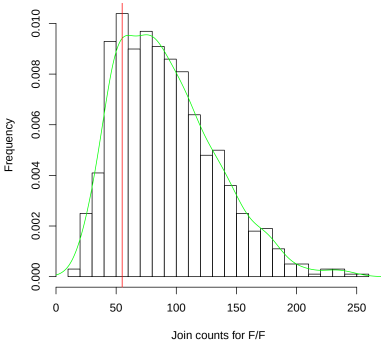
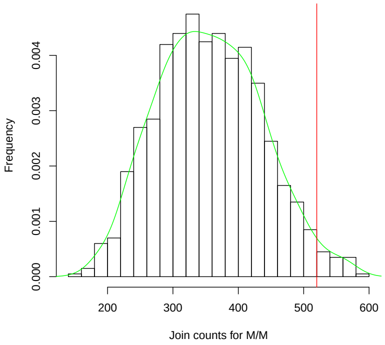
Empirical distribution with Monte Carlo approach ($P = 1000$)



JC_F



JC_M

A toy example: “Les Misérables”

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Estimated p-value and conclusion

Gender	Join count value	Large	Small
F	55	0.7932 (NS)	0.2068 (NS)
M	520	0.0224 (**)	0.9755 (NS)

Men have a tendency to interact with other men rather than with women in “Les Misérables” whereas women don’t have a specific way to be related according to gender.

Plan

Alternative approach: graph visualization

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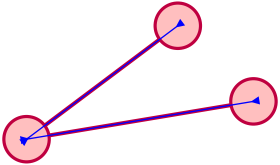

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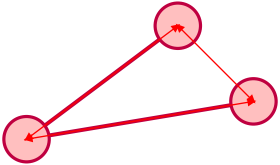

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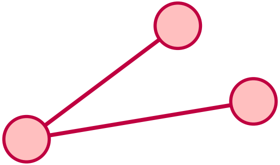

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iterative algorithm until stabilization of the nodes positions.

State-of-the art: clustered graph visualization

Main idea: Labels can be seen as a clustering ⇒ use visualization approach that allows **the nodes with the same labels to be displayed close to each others**.

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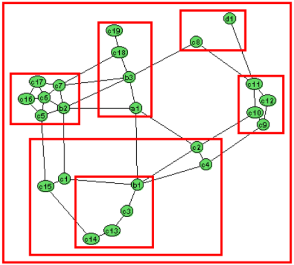
- **Modified force directed placement algorithms [Bourqui et al., 2007, Eades and Feng, 1996, Eades and Huang, 2000, Truong et al., 2007]**: integrate additional constraints into forces or constrain vertices to be displayed in a given zone, according to their clusters;



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- **Modified force directed placement algorithms**
  The graph can be displayed in a **simplified way** (one “meta-node” per cluster) as in **[Rossi and Villa-Vialaneix, 2011]**.

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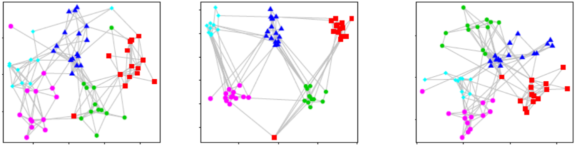
- **Modified force directed placement algorithms**
- **Use of latent variables [Bouveyron et al., 2009]**



(a) Usual latent space (b) Supervised latent space (SL1) (c) Supervised latent space (SL2)

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- **Use of latent variables**

All these approaches:

- only consider the node's label and do not use the neighbors' labels;
- do not deal with multiple labels.

Plan

PCA based on neighbors' labels distribution

Denote:

- E the disjunctive encoding of nodes' labels

$$E_{ij} = \begin{cases} 1 & \text{if } c_j \in \mathcal{C}(x_i) \\ 0 & \text{if } c_j \notin \mathcal{C}(x_i) \end{cases}$$

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- P_l, the labels distribution among the neighbors:

$$P_l = D^{-1} W E$$

where $D = \mathrm{Diag}(d_1, \ldots, d_n)$ with d_i degree of node x_i.

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Display the graph with the coordinates in the **Weighted PCA** of P_l where columns are weighted by $\frac{n_j}{n}$ with $n_j = \left|\{x_i :\ c_j \in \mathcal{C}(x_i)\}\right|$.

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Remark: This choice is similar to the use of the χ^2 metric:

$$\delta(p_i, p_{i'}) = \sum_c \frac{n}{n_c} \left(\frac{n_{ic}}{d_i} - \frac{n_{i'c}}{d_{i'}} \right)^2$$

Kernel based approach

Previous method **drawbacks**:

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Alternative approach: Use a diffusion process by means of the **heat kernel**

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Heat kernel features:

- has a simple interpretation regarding a diffusion process along the edges of the graph;
- can be viewed as a dot product between nodes in an embedding space:

$$K_{ij}^\beta \equiv K^\beta(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle_{\mathcal{K}^\beta}$$

Kernel PCA for labeled graph visualization

- Use $K^\beta E$ instead of P_l to represent the labels distribution among the neighbors (the node's label is used):

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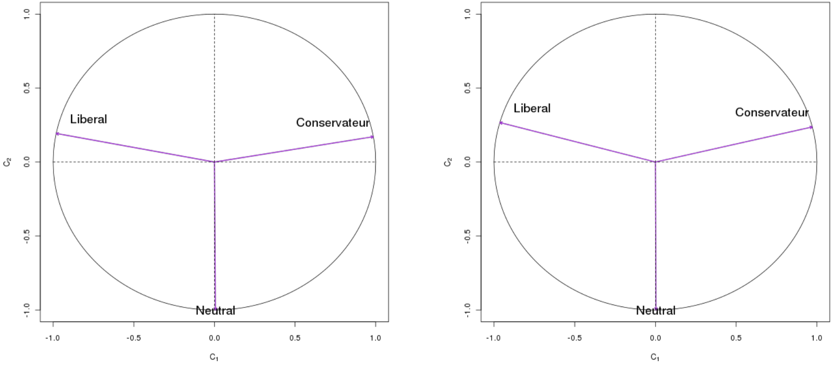
Various β will provide various representation: small β favor direct neighbors.

Plan

Polbooks

Co-purchase network: nodes are books sold by “Amazon” and are labeled according to the political orientation of the book.

Labels representation



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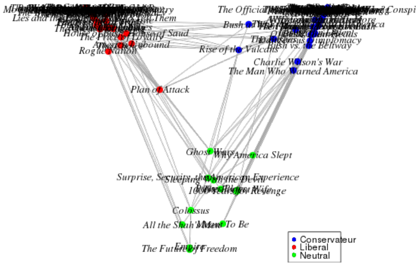

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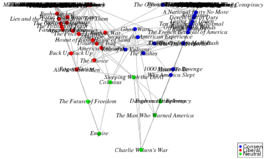
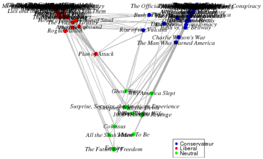




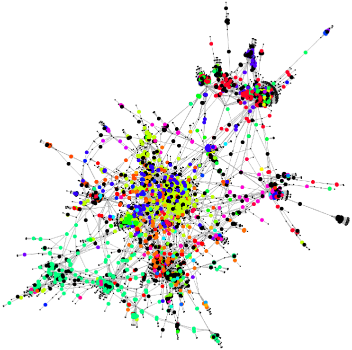
Main conclusions:

- Strong relations between labels and graph structure: nodes with the same labels also have the same labels distribution among their respective neighbors;
- Provide a **more subtle interpretation** of the book’s political orientation (ex: “World of Vulcain” is conservative but close to liberal)
- Differences between the two representations (ex: “Plan of attack” is frequently co-purchased with liberal books that are themselves frequently co-purchased with non-liberal books)

Medieval

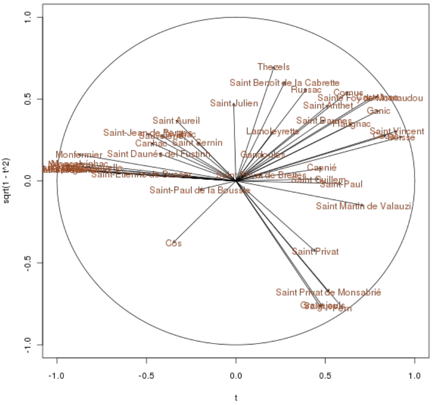
Bipartite graph

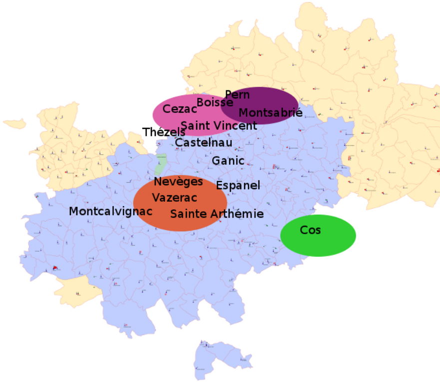
- nodes: transactions and individuals (3 918 nodes)
- edges: an individual is directly involved in a transaction (6 455 edges)
- labels (transactions only): location (parish)



Medieval

PCA applied on the **individuals** only (projected network) based on the location distribution among transactions (multiple labels).





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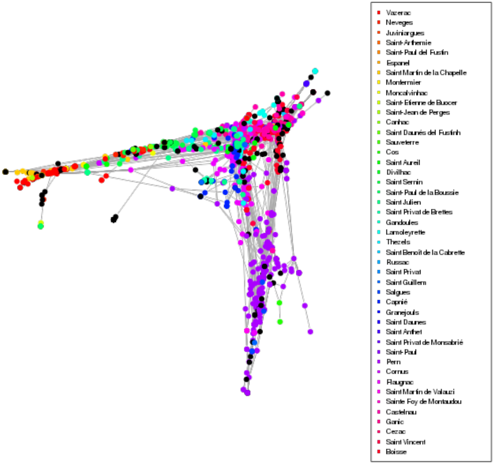
Vazerac
Neveges
Juviniargues
Saint-Arthemie
Saint-Paul del Fustin
Espanel
Saint Martin de la Chapelle
Montermier
Moncalvinhac
Saint-Etienne de Bucer
Saint-Jean de Perges
Canhac
Saint Daunès del Fustinh
Sauveterre
Cos
Saint Aureil
Divilhac
Saint Sernin
Saint-Paul de la Boussie
Saint Julien
Saint Privat de Brettes
Gandoules
Lamoleyrette
Thezels
Saint Benoît de la Cabrette
Russac
Saint Privat
Saint Guillem
Salgues
Caprié
Granejouls
Saint Daunes
Saint Anthet
Saint Privat de Monsabrié
Saint-Paul
Pern
Cornus
Flaugnac
Saint Martin de Valauzi
Sainte Foy de Montaudou
Castelnau
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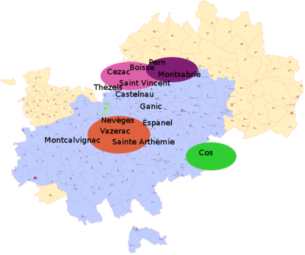
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Montcalvignac
Sainte Arthémie
Cos

Any questions?...

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